hey

Unit 2 Test Prep Review Fall 25

1. Using the following exponential function:

$C \cdot a^x$

f(x) = 5(4/5)

a. What is the y-intercept (initial value)? C 5

b. Is this a growth or decay exponential? $\frac{4}{5} = 0.8 \rightarrow$ Decay

c. Growth factor? 0.8 or $\frac{4}{5}$

d. Growth rate? $a = 1 + r$ so, $r = a - 1$ $\frac{4}{5} - 1 = -\frac{1}{5}$ or -0.2

e. Determine f(4). Round to 3 decimal places.

$f(4) = 5\left(\frac{4}{5}\right)^4 = \boxed{2.048}$

2. A car initially costs $32,000 and depreciates at a rate of 6% per year.

C r is negative

6% DP.

$-0.06 = r$

a. Write an equation for the car's value, V(t), where t is in years.

$V(t) = Ca^x \rightarrow V(t) = 32000(0.94)^t$

$a = 1 + r \rightarrow 1 + -0.06 = 0.94$

b. Rewrite the equation to model the car's monthly depreciation.

$V\left(\frac{1}{12}\right) = 32000(0.94)^{1/12} = \$31,835.42$

d. Determine the car's value after 3 years.

$V(3) = 32000(0.94)^3 = \$26,578.69$

3. The growth factor for a population over 6 months was 3.6. Determine the monthly growth factor.

1 period = 6 months = 3.6

$\frac{1}{6}$ period = 1 month = $(3.6)^{\frac{1}{6}} = \boxed{1.24}$

4. Write the Exponential model for the table below:

$\frac{\cancel{405}a^2}{\cancel{405}} = \frac{180}{405}$

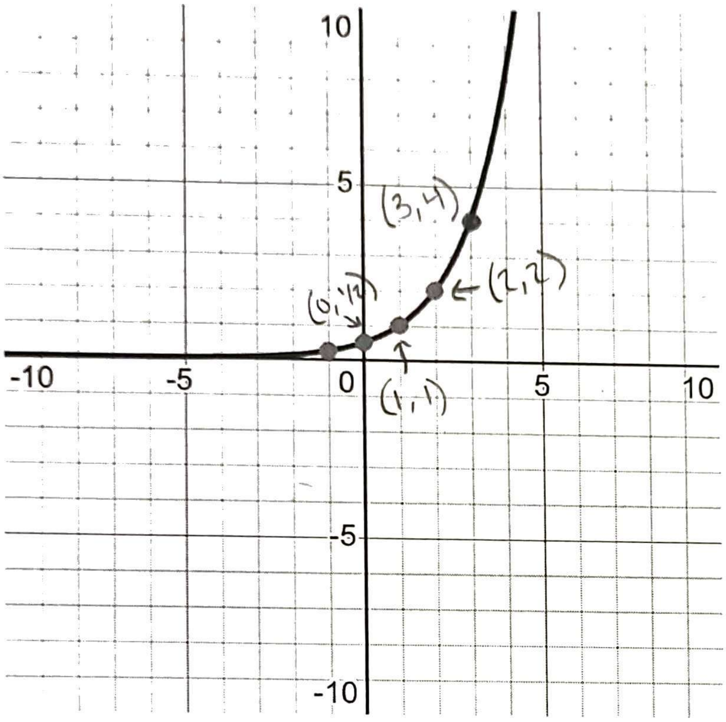
$(a^2)^{\frac{1}{2}} = (0.\overline{4444})^{\frac{1}{2}} = 0.\overline{66}$ or $\frac{2}{3}$

+2 +2 +2

x	-2	0	2	4
f(x)	405	C = 180	80	36

$f(x) = Ca^x \rightarrow \boxed{f(x) = 180\left(\frac{2}{3}\right)^x}$

5. Write the formula for the given exponential graph.



X	Y
0	1/2 or 0.5
1	1
2	2
3	4

$\frac{2a}{2} = \frac{4}{2}$

$a = 2$

$f(x) = Ca^x \rightarrow \boxed{f(x) = \frac{1}{2}(2)^x}$

6. The table below represents a relationship between x and h(x):

x	-2	-1	0	1	2
h(x)	2.25	3	C = 4	5.33	7.11

a. Is the function linear or exponential? Determine using constant and percent rate of change.

Exponential

b. Determine the equation that models this table.

$h(x) = Ca^x \rightarrow \boxed{h(x) = 4(1.\overline{33})^x}$

$r = 0.\overline{33}$

$a = 1 + r = 1.\overline{33}$

Constant $(y_2 - y_1)$

$3 - 2.25 = .75$

$4 - 3 = 1$

$5.\overline{33} - 4 = 1.\overline{33}$

Not Linear

Percent rate $\left(\frac{y_2 - y_1}{y_1}\right)$

$\frac{3 - 2.25}{2.25} = 0.\overline{33}$

$\frac{4 - 3}{3} = 0.\overline{33}$

$\frac{5.\overline{33} - 4}{4} = 0.\overline{33}$

Exponential ✓

Constant $(y_2 - y_1)$

$6 - 4 = 2$
$8 - 6 = 2$ } Linear ✓
$10 - 8 = 2$

7. The table below represents a relationship between x and f(x):

x	-2	-1	0	1	2
f(x)	4	6	$b_0 =$ 8	10	12

a. Is the function linear or exponential? Determine using constant and percent rate of change.

Linear

b. Determine the equation that models this table.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$f(x) = mx + b \longrightarrow f(x) = 2x + 8$

$m = \frac{12 - 10}{2 - 1} = \frac{2}{1}$

8. Using the table below, determine if the data set represents a linear or exponential function.

x	f(x)	Average Rate of Change $(y_2 - y_1)$	Percent Rate of Change $\rightarrow \frac{y_2 - y_1}{y_1}$
-2	18000		
-1	12000	$12000 - 18000 = -6000$	$\frac{-6000}{18000} = -.\overline{33} \rightarrow 33.\overline{3}\%$
0	8000	$8000 - 12000 = -4000$	$\frac{-4000}{12000} = -.\overline{33} \rightarrow 33.\overline{3}\%$
1	5333	$5333 - 8000 = -2667$	$\frac{-2667}{8000} = -.\overline{33} \rightarrow 33.\overline{3}\%$

Not Linear

Exponential ✓

9. The population of fish in a pond is modeled by:

$$f(t) = \frac{600}{1 + 5 * 2^{-t}}$$

$-t\log(2^{t}) = \log(0.1)$

a. What is the carrying capacity?

600

b. What was the initial population?
$t = 0$

$f(0) = \frac{600}{1 + 5 \cdot 2^{-0}} = \boxed{100}$

c. Find the population after 4 years.
$t = 4$

$f(4) = \frac{600}{1 + 5 \cdot 2^{-4}} = \boxed{457}$

d. How many years will it take to reach 400 fish?
$f(t)$

$(1 + 5 \cdot 2^{-t})400 = \frac{600}{1 + 5 \cdot 2^{-t}}\cancel{(1 + 5 \cdot 2^{-t})}$

$\frac{\cancel{400}(1 + 5 \cdot 2^{-t})}{\cancel{400}} = \frac{600}{400}$

$\cancel{1} + 5 \cdot 2^{-t} = 1.5$
$-1 \qquad -1$

$\frac{\cancel{5} \cdot 2^{-t}}{\cancel{5}} = \frac{0.5}{5}$

$2^{-t} = 0.1$

$\frac{\log_2(0.1)}{-1} = \frac{\cancel{-}t}{\cancel{-1}}$

$\boxed{3 \text{ years}}$

P $\quad 0.046 = r$

10. A principal amount of $800 is invested at an annual interest rate of 4.6%.

a. Find the total amount after 8 years if compounded quarterly.

t $\quad 4$

$$A(t) = 800\left(1 + \frac{0.046}{4}\right)^{4(8)} = \boxed{\$1,153.45}$$

b. Find the total amount after 8 years if compounded continuously.

e

$$A(t) = 800e^{0.046(8)} = \boxed{\$1,155.87}$$

11. Rewrite the exponential model y = 6.2(5/8)^x as an equivalent model with base e.

(in the form y = Ce^{rx}). Round r to 3 decimal places.

$y = C\boxed{a}^x \rightarrow y = C\boxed{e}^{rx}$ $\qquad y = 6.2\left(\frac{5}{8}\right)^x$

$e^r = a$

$\ln(e^r) = \ln(a)$ $\quad r = \ln\left(\frac{5}{8}\right)$

$r = \ln(a)$ $\quad r = -0.47$

$$\boxed{y = 6.2e^{-0.47x}}$$

12. Expand and simplify: $\log_4\left(\frac{16x^3\sqrt[3]{y}}{z^2}\right)$

$$\log_4(16) + \log_4(x) + \frac{1}{3}\log_4(y) - 2\log_4(z)$$

13. Condense and simplify: $3\ln(x) - 4\ln(z) + 2\ln(y) - 5\ln(w)$

$$\underbrace{\ln(x^3)}_{T} - \underbrace{\ln(z^4)}_{B} + \underbrace{\ln(y^2)}_{T} - \underbrace{\ln(w^5)}_{B}$$

$$\boxed{\ln\left(\frac{x^3y^2}{z^4w^5}\right)}$$

14. A bacteria culture starts with 40 cells and increases at a rate of 35% per hour.

C $\quad .35 \quad r = 0.35$

a. Write an exponential model for this growth.

$a = 1 + r = 1 + 0.35 = 1.35$

$$f(x) = Ca^x \rightarrow \boxed{f(x) = 40(1.35)^x}$$

b. Predict the number of bacteria after 6 hours.

x

$f(6) = 140(1.35)^6 = \boxed{242.14}$ or $\boxed{242}$

15. A dose of 100 mg of a drug is administered to a patient. Each hour, 20% of the drug leaves the body.

C $r = -0.20$ → r is negative

a. Write an exponential decay model for the amount remaining after x hours.

$f(x) = 100(0.8)^x$

$a = 1 + r$

$a = 1 + -0.20 = 0.8$

b. Predict the amount after 5 hours.

x

$f(5) = 100(0.8)^5 = \boxed{32.768}$

c. Predict when the amount will reach 25 mg.

$f(x)$

$\frac{25}{100} = \frac{\cancel{100}(0.8)^x}{\cancel{100}}$

$0.25 = 0.8^x \rightarrow \log_{0.8}(0.25) = x \rightarrow \boxed{6.21 = x}$

16. The function S(x) = 28 + 2.4log(x + 1) models ocean salinity x meters deep.

a. Find the depth where the salinity equals 31.

$S(x)$

$31 = \cancel{28} + 2.4\log(x+1)$

$-28 \quad -\cancel{28}$

$\frac{3}{2.4} = \frac{\cancel{2.4}\log(x+1)}{\cancel{2.4}}$

$1.25 = \log(x+1)$

$10^{1.25} = x + \cancel{1}$

$-1 \quad \cancel{-1}$

$\boxed{16.78 = x}$

b. Find the salinity at a depth of 600 meters.

x

$S(600) = 28 + 2.4\log(600+1)$

$\boxed{S(600) = 34.67}$